A Stochastic Model Predictive Control Approach for Series Hybrid Electric Power Management

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Hybrid Electric Vehicles

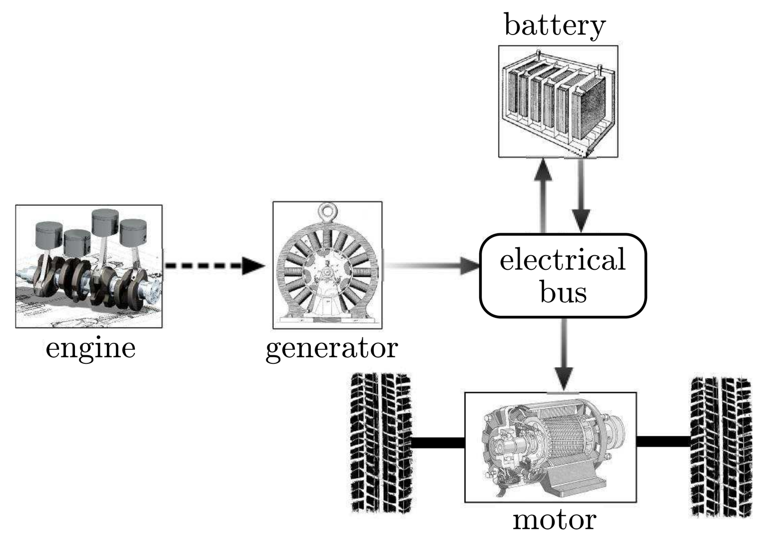
Motivation: Rising fuel prices and tightening emission regulations have resulted in an increasing need for advanced powertrain systems and **systematic model-based control** approaches.

Solution: **Hybridization** of ICE with electric motors.

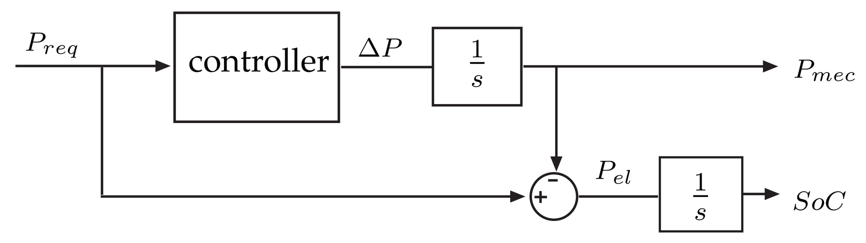
Challenges: Cope with tight specification, high performance required, high fidelity model, complex system. Extend previous approach based on deterministic model predictive control [1].

Powertrain Model

Consider the powertrain configuration of a **series HEV**:



A simplified powertrain model is obtained by considering the current **power demand** $P_{req}(k)$ as a measured disturbance:



The **electrical battery** is the only component described by a dynamical model:

$$SoC(k+1) = SoC(k) - KTP_{el}(k)$$

Under the assumption that the current power demand is always satisfied, the powertrain **linear system** can be cast as:

$$\begin{cases} x(k+1) = Ax(k) + B_1\Delta P(k) + B_2 P_{req}(k) \\ P_{el}(k) = Cx(k) + D_1\Delta P(k) + D_2 P_{req}(k) \end{cases}$$

$$\begin{aligned} x(k) &= [SoC(k)\ P_{mec}(k-1)]' \\ u(k) &= [\Delta P(k)\ P_{req}(k)]' \\ y(k) &= P_{el}(k) \end{aligned}$$

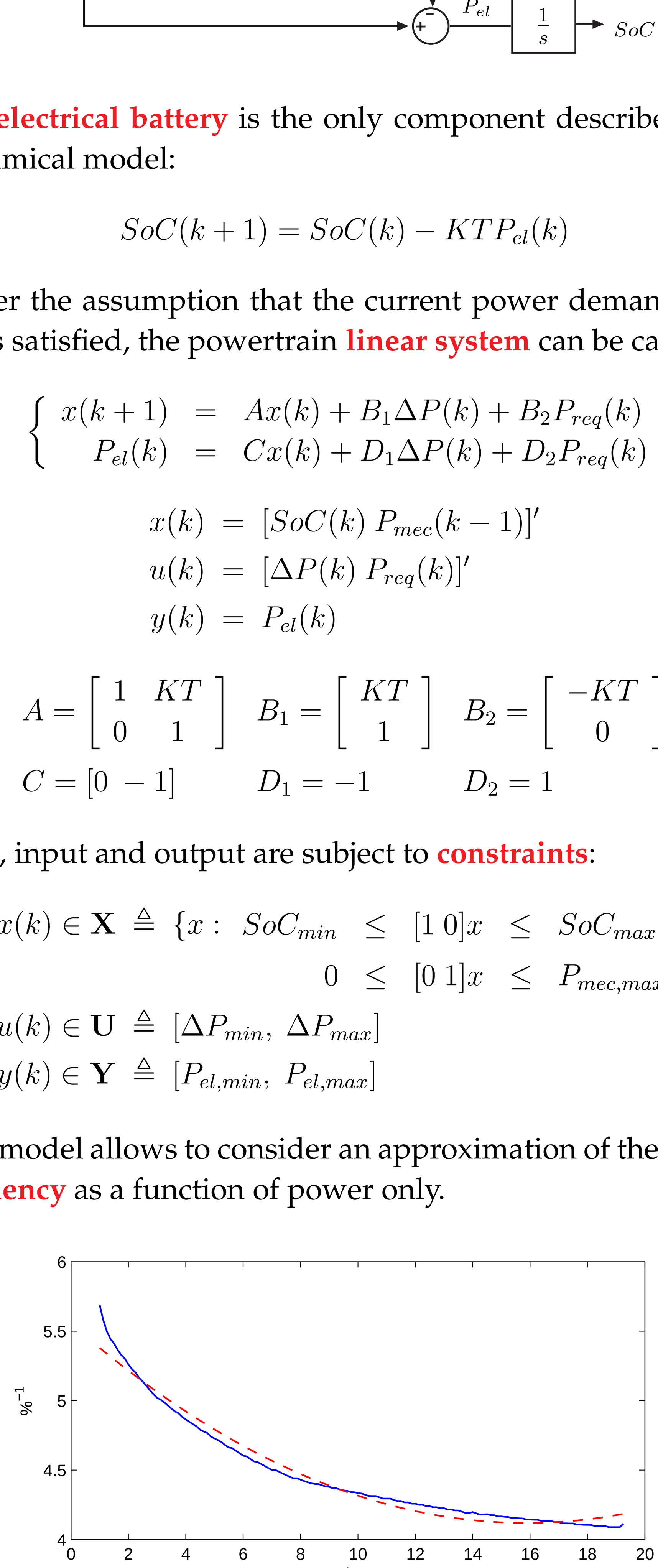
$$A = \begin{bmatrix} 1 & KT \\ 0 & 1 \end{bmatrix} \quad B_1 = \begin{bmatrix} KT \\ 1 \end{bmatrix} \quad B_2 = \begin{bmatrix} -KT \\ 0 \end{bmatrix}$$

$$C = [0\ -1] \quad D_1 = -1 \quad D_2 = 1$$

State, input and output are subject to **constraints**:

$$\begin{aligned} x(k) \in \mathbf{X} &\triangleq \{x :\ SoC_{min} \le [1\ 0]x \le SoC_{max} \\ &\qquad\quad 0 \le [0\ 1]x \le P_{mec,max}\ \} \\ u(k) \in \mathbf{U} &\triangleq [\Delta P_{min},\ \Delta P_{max}] \\ y(k) \in \mathbf{Y} &\triangleq [P_{el,min},\ P_{el,max}] \end{aligned}$$

This model allows to consider an approximation of the engine **efficiency** as a function of power only.



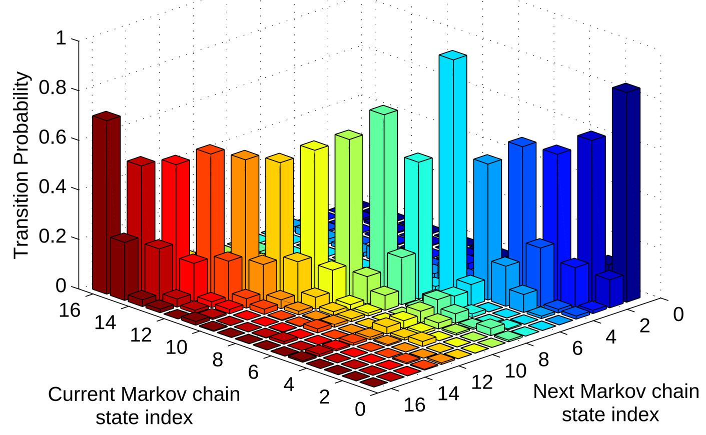
Legend: Inverted efficiency η^{-1}; Quadratic approximation $J_{\eta^{-1}}$

Stochastic Modeling of Power Request

Objective: Design a stochastic model to **estimate** future power requests by exploiting available statistical information, without assuming a priori knowledge of the drive cycle.

Technique: Use a **Markov chain** tuned over several standard drive cycles. The Markov chain is defined by a set of states $\{z_1, z_2, \ldots, z_n\}$ and a transition matrix $T \in \mathbb{R}^{n\times n}$ whose elements t_{ij} are

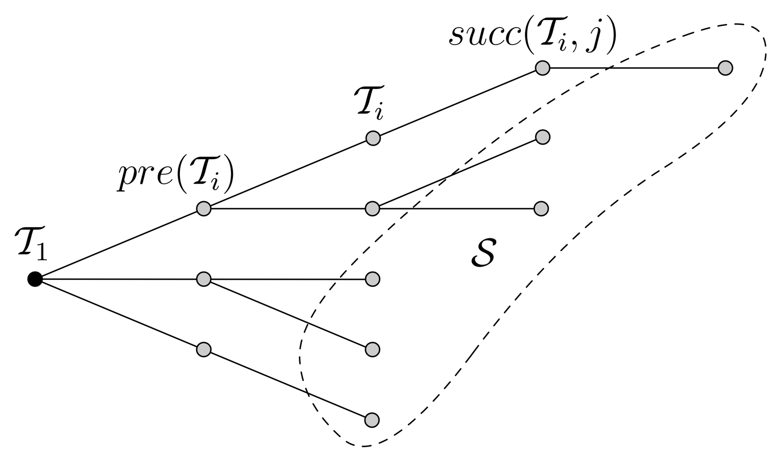
$$t_{ij} = \mathbf{P}[P_{req}(k+1) = z_j | P_{req}(k) = z_i]$$



Stochastic MPC Design

Main idea: We adopt the **stochastic MPC** (SMPC) approach presented in [2] based on scenario enumeration, which exploits ideas from multi-stage stochastic optimization. The knowledge of the disturbance model is used by the SMPC to possibly improve the closed-loop performance of the controlled system with respect to a standard deterministic MPC algorithm.

Optimization tree design: The SMPC problem formulation is based on a **maximum likelihood** approach, where at every time-step an optimization tree is built using the updated information on the system state and on the Markov chain. Each node of the tree represents a predicted state which is taken into account in the optimization problem.



Starting from the root node, which is defined by $x(k)$ and $P_{req}(k)$, a list of candidate nodes is generated by considering all the possible future Markov states $P_{req}(k+1|k)$, together with their realization probability. Then, the node with maximum probability is added to the tree. This procedure is repeated iteratively, until a desired number of nodes N is reached.

Control problem formulation: The SMPC problem at time k is formulated as

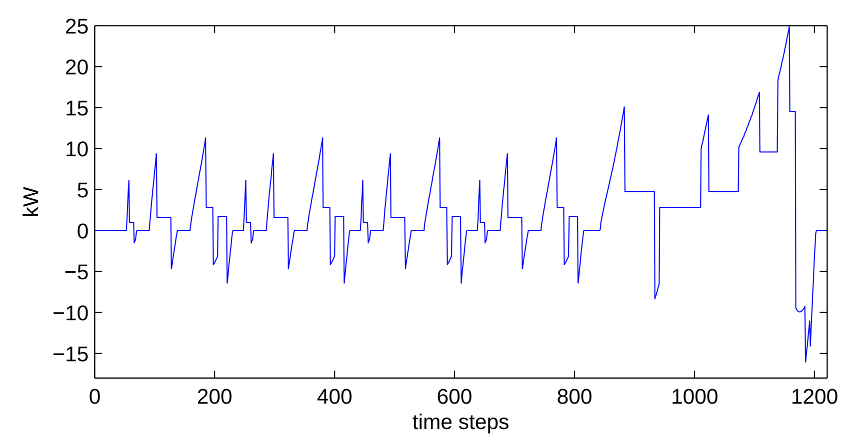
$$\begin{aligned} \min_{\{\Delta P_i\}} \quad & \sum_{i\in\mathcal{T}\setminus\{\mathcal{T}_1\}} \pi_i (x_i - x_{ref})' \begin{bmatrix} Q_{SoC} & 0 \\ 0 & \phi Q_J \end{bmatrix} (x_i - x_{ref}) + \\ & + \sum_{j\in\mathcal{T}\setminus\mathcal{S}} \pi_j Q_P \Delta P_j^2 && \text{(1a)} \\ \text{s.t.} \quad & x_1 = x(k), && \text{(1b)} \\ & P_{req,1} = P_{req}(k), && \text{(1c)} \\ & x_i = Ax_{pre(i)} + B_1\Delta P_{pre(i)} + B_2 P_{req,pre(i)}, \\ & \quad \forall i \in \mathcal{T}\setminus\{\mathcal{T}_1\}, && \text{(1d)} \\ & P_{el,i} = Cx_i + D_1\Delta P_i + D_2 P_{req,i}, \\ & \quad \forall i \in \mathcal{T}\setminus\mathcal{S}, && \text{(1e)} \\ & x_i \in \mathbf{X},\ \forall i \in \mathcal{T}\setminus\{\mathcal{T}_1\}, && \text{(1f)} \\ & \Delta P_i \in \mathbf{U},\ \forall i \in \mathcal{T}\setminus\mathcal{S}, && \text{(1g)} \\ & P_{el,i} \in \mathbf{Y},\ \forall i \in \mathcal{T}\setminus\mathcal{S}, && \text{(1h)} \end{aligned}$$

where $x_{ref} = [SoC_{ref}\ P_{mec,ref}]'$. Problem (1) is a QP.
By imposing $P_{mec,ref} = P^*$ we have a term in the cost function to **maximize** an approximation of the engine efficiency η. The power demands associated with the prediction nodes $P_{req,i}$, $i > 1$, are obtained from the Markov chain model.

Simulation Results

The closed-loop behavior of the HEV in closed loop with the SMPC has been evaluated in simulations on the NEDC drive cycle by using a quasi-static model provided by Ford Motor Company.

NEDC drive cycle

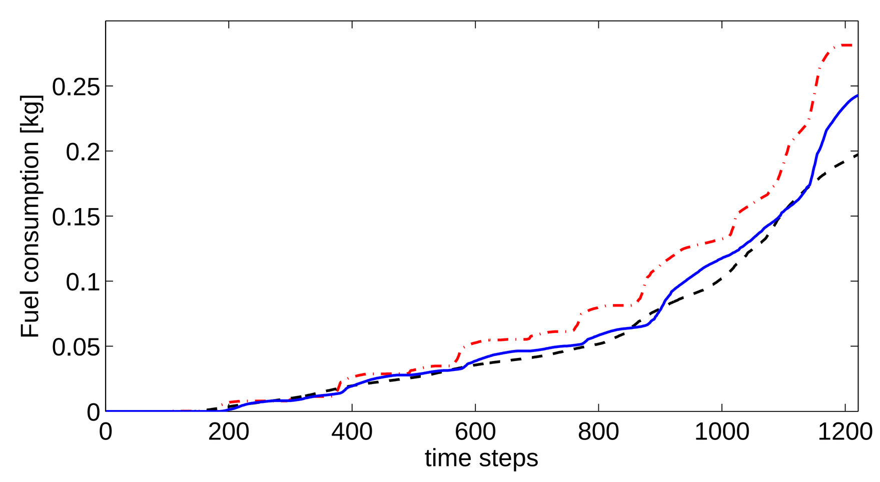


SMPC designed with a 100-nodes optimization tree is compared with two controllers: a **frozen-time MPC** (FTMPC), which exploits no information on the future, and a **prescient MPC** (PMPC), which exploits a priori knowledge of the requested power demand for a given future horizon window.

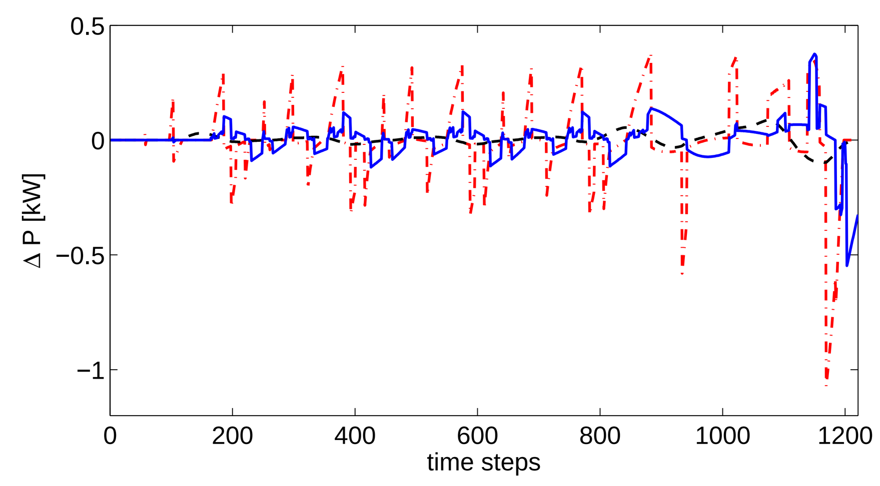
Closed-loop numerical results

	$\|\|\Delta P\|\|$	fuel cons. [kg]	fuel improv. [%]
FTMPC	5.7906	0.281	0
SMPC	3.1236	0.243	13.5
PMPC	0.8581	0.197	29.8

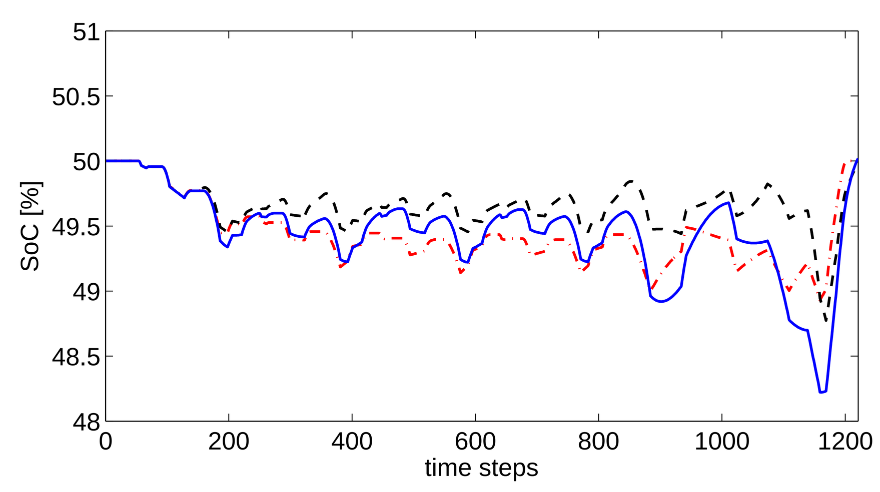
Fuel consumption



Variation of mechanical power



State of charge of the battery



Legend: SMPC, FTMPC, PMPC.

References

[1] G. Ripaccioli, A. Bemporad, F. Assadian, C. Dextreit, S. Di Cairano, and I. Kolmanovsky, "Hybrid Modeling, Identification, and Predictive Control: An Application to Hybrid Electric Vehicle Energy Management," *Hybrid Systems: Computation and Control*, pp. 321–335, 2009.

[2] D. Bernardini and A. Bemporad, "Scenario-based model predictive control of stochastic constrained linear systems," in *Proc. 48th IEEE Conf. on Decision and Control*, 2009, to appear.