Robust Analysis Techniques for Clearance Of Flight Control Laws

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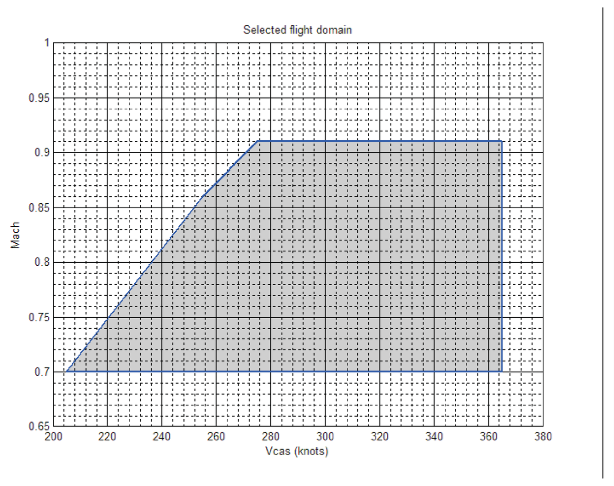
INTRODUCTION

Clearance of flight control laws is an integral part of the certification of the flyability of an aircraft, and it is codified in terms of criteria regarding the stability (structural and dynamical) and performance (manouvrability, comfort, etc.) which the aircraft has to satisfy for all the possible flight configurations. A commonly practiced approach in industrial aircraft clearance is the grid-based approach: a finite set of grid points are selected within the flight/uncertainty domain and the clearance criterion of interest is tested for each point. The main drawbacks of this approach are the high computational times required (dependent on the density of the grid), which also affect the design procedure costs, and the fact that only grid points are actually cleared and not the entire domain, thus compromising the reliability of the clearance process.



THE EUROPEAN PROJECT COFCLUO

Motivated by the idea that clearance problems can be formulated as robustness analysis problems, the European Project COFCLUO (Clearance Of Flight Control Laws Using Optimization) within the Sixth Framework Programme, has the aim to improve the reliability of certifications and to reduce global design costs by exploiting results from optimization and robust control theory. In particular, Lyapunov stability results involving parameter-dependent candidate functions have been used to assess the stability of an aircraft respresented by uncertainty models called LFRs (Linear Fractional Representations).
COFCLUO is a three years project (2007-2010), and involves six European research groups:

- LiU, Linköpings Universitet, Linköping (Sweden);
- AIRBUS France S.A.S., Toulose (France);
- ONERA, Office National d'Etudes et de Recherches Aerospatiales, Toulose (France);
- DLR, Deutsches zentrum für Luft- und Raumfahrt, Münich (Germany);
- FOI, Swedish Defence Research Agency, Stockholm (Sweden);
- UNISI, Università degli studi di Siena, Siena (Italy).

AIRBUS, ONERA, and DLR, have been in charge of generating the mathematical representation of the aircraft; UNISI, LiU, FOI, ONERA, and DLR, of developing the clearance techniques.

UNCERTAINTY MODELS

The controlled aircraft is described by the LFR

$$\dot{x}(t) = \mathbf{A}(\theta)x(t) = (A + B\Delta(\theta)(I - D\Delta(\theta))^{-1}C), \qquad (1)$$

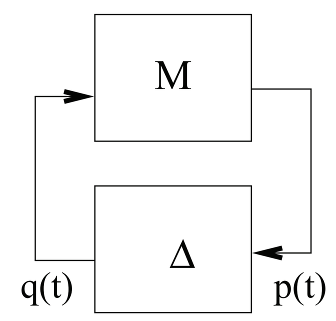
where $x \in \mathbb{R}^n$ is the state vector, $\mathbf{A}(\theta)$ is a rational function of the uncertain parameter vector $\theta \in \mathbb{R}^{n_\theta}$ ($Mach$, $Vcas$, PL, OT, CT, Xcg) [1], and

$$\Delta(\theta) = \mathbf{diag}(\theta_1 I_{s_1}, \dots, \theta_{n_\theta} I_{s_{n_\theta}}), \qquad (2)$$

θ_i denotes the i-th component of θ. The parameters θ_i represent flight envelope parameters (like Mach number and air speed) or uncertainties in the aircraft configuration (tank fullness, center of gravity). An equivalent representation of the system (1)-(2) is given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bq(t) \\ p(t) = Cx(t) + Dq(t) \\ q(t) = \Delta(\theta)p(t), \end{cases} \qquad (3)$$

where $q,\ p \in \mathbb{R}^d$, with $d = \sum_{i=1}^{n_\theta} s_i$. This is the $M - \Delta$ structure, a standard paradigm of robust control theory. A, B, C, D are real matrices of appropriate dimensions, with Hurwitz A. The uncertain parameter vector θ belongs to a iper-rectangular set Θ, representing the region of interest. The vertices of Θ are denoted by Ver[Θ].



The uncertain parameters are assumed to be constant, i.e. $\dot{\theta} = 0$.

ROBUST STABILITY PROBLEM

A number of clearance problems can be cast as robust stability analysis problems for LFR models, by applying Lyapunov stability theory.
Basic idea:
The global robust asymptotic stability of the equilibrium $x_0 = 0$ can be deduced from the existence of a Lyapunov function $V(x;\theta) : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}$ that satisfies:

$$\begin{cases} V(x;\theta) > 0, & \forall x \in \mathbb{R}^n_0,\ \forall \theta \in \Theta \\ \dot{V}(x;\theta) < 0, & \forall x \in \mathbb{R}^n_0,\ \forall \theta \in \Theta,\ \forall \dot{\theta} \in \Phi \end{cases} \qquad (4)$$

where $\mathbb{R}^n_0 = \mathbb{R}^n \setminus \{0\}$.
Unfortunatly, the resulting problems are in general not convex and hence intractable.
However, it is often possibile to derive convex relaxations of such problems, which provide sufficient conditions for robust stability. In most cases, the convex relaxations are formulated in terms of Linear Matrix Inequalities (LMIs). An LMI has the form

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i > 0, \qquad (5)$$

where $x \in \mathbb{R}^m$ is the variable and the symmetric matrices $F_i = F_i' \in \mathbb{R}^{n\times n}$, $i = 0, ..., m$, are given.Finding an x satisfying (5) is an SDP (Semi-Definite Programming) problem that can be tackled by efficient optimization tools.

A FAMILY OF CONVEX RELAXATIONS

A sufficient condition for robust stability of system (3) involving a candidate Lyapunov function whose dependence on the uncertain parameter is multi-affine, i.e.

$$V(x) = x^T P(\theta) x = x^T \left(P_0 + \sum_{j=1}^{n_\theta} \theta_j P_j + \sum_{i=1}^{n_\theta} \sum_{j=i+1}^{n_\theta} \theta_i \theta_j P_{ij} + \dots \right) x$$

can be stated as follows:
Proposition (FD-cμ) *System* (3) *is robustly asymptotically stable for all $\theta \in \Theta$ if there exist matrices $G \in \mathbb{R}^{d\times n}$, $H \in \mathbb{R}^{d\times d}$, and $P(\theta) \in \mathbb{R}^{n\times n}$ such that $\forall \theta \in$ Ver[Θ]*

$$\begin{cases} P(\theta) > 0 \\ \begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} + \begin{bmatrix} C(\theta)^T G + G^T C(\theta) & C(\theta)^T H + G^T D(\theta) \\ H^T C(\theta) + D(\theta)^T G & D(\theta)^T H + H^T D(\theta) \end{bmatrix} < 0, \end{cases}$$

where $C(\theta) = \Delta(\theta) C$ *and* $D(\theta) = (\Delta(\theta) D - I)$.

This condition could be computational demanding to be solved since it is composed by v constraints of dimension $n + d$, and v constraints of dimension n, with a total number of free variables $n_{var} = nd + d^2 + 2^{n_\theta}\binom{n(n+1)}{2}$, which in the standard application is very high. In a standard SDP solver, the time spent in the search direction computation is $O(n_{var}(n+d)^3 + n_{var}^2(n+d)^2)$.
In [1] it has been shown how it is possible to reduce the required computational burden (at the price of introducing additional conservatism) by:

- simplifying the structure of the candidate Lyapunov function: common (clf), affinely parameter dependent (apdlf),
- imposing structure on the "so-called" multipliers G and H: FD-cdμ,DS, DS-dS, WBQ-dM .

GENERATION OF THE LFRS

Several LFRs of the closed-loop longitudinal dynamics of a civil aircraft have been generated by ONERA and DLR [2], from a set of linear aeroelastic models provided by AIRBUS, depending on the mass configuration (expressed in terms of fullness of two fuel tanks and a payload) and the trim flight point (characterized by Mach number and conventional air speed) derived from a flutter representation of the aircraft:

Model	n	d	$\theta_1, s1$	$\theta_2, s2$	$\theta_3, s3$
C	20	16	C, 16	–	–
OC	20	50	C, 26	O, 24	–
POC	20	79	C, 42	O, 24	P, 13
MV	20	54	M, 26	V, 28	–

CLEARANCE RESULTS

In order to tackle the robust stability problem related to the aeroelastic stability clearance criterion, it has been used the so-called Partitioning Approach [1]. In this approach, the idea is to progressively partition the flight/uncertainty domain Θ into rectangular regions, and then, apply the robustness analysis conditions to each region. The size of each region is reduced until the whole domain is cleared, or the predefined minimum size is reached.

C MODEL

Method	Rate	OPS	Time (sec)
DS (clf)	1	1	9.78
DS (apdlf)	1	1	10.97
DS-dS (clf)	1	3	10.15
DS-dS (apdlf)	1	1	4.15
FD-cμ (clf)	1	1	5.81
FD-cμ (apdlf)	1	1	8.77
FD-cdμ (clf)	1	3	8.43
FD-cdμ (apdlf)	1	1	4.93

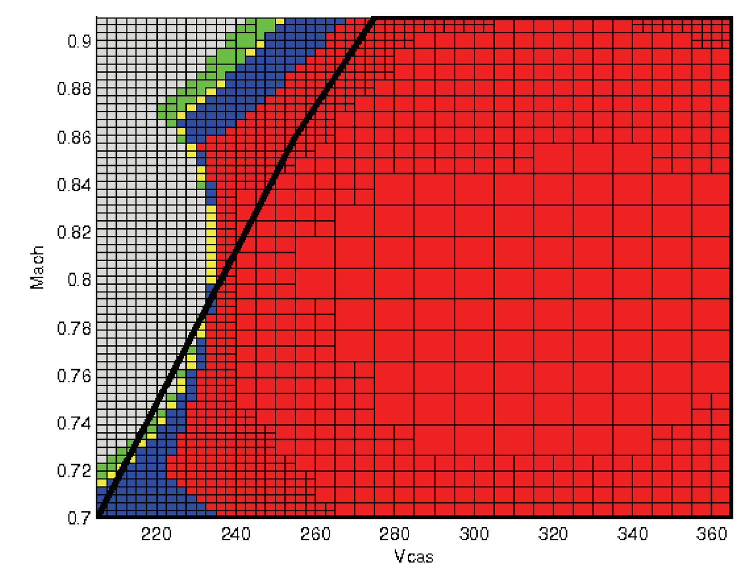
OC MODEL

Method	Rate	OPS	Time (hours)
DS-dS (clf)	1	73	0.4424
DS-dS (apdlf)	1	41	0.5311
FD-cμ (clf)	1	33	2.9252
FD-cμ (apdlf)	1	1	0.1032
FD-cdμ (clf)	1	85	0.3668
FD-cdμ (apdlf)	1	49	0.5308

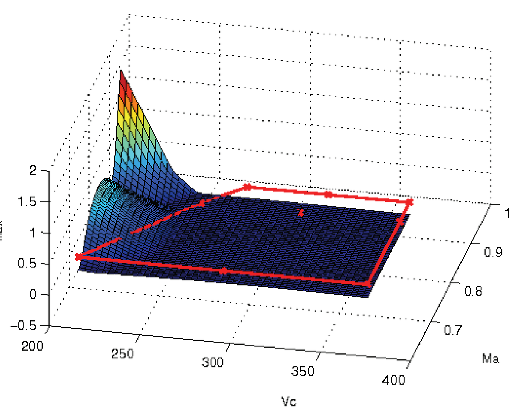
POC MODEL

Method	Rate	OPS	Time (hours)
DS-dS (clf)	1	993	33.7136
FD-cμ (clf)	1	105	142.6362

MV MODEL



Clearance of MV model by progressive tiling: green+yellow+blue+red (FD-cμ), yellow+blue+red (DS-dS), blue+red (FD-cdμ), red (WB-dN).



Closed-loop longitudinal MV model: max{Real(λ)}.

Method	Rate	OPS	Time (hours)
DS-dS (clf)	0.976	788	7.59
FD-cμ (clf)	0.998	124	17.64
FD-cdμ (clf)	0.969	832	5.34
WBQ-dM	0.920	1576	5.64

References

[1] A. Garulli, A. Masi, S. Paoletti, and E. Turkoglu, "Clearance of flight control laws via parameter-dependent Lyapunov functions," in *6th IFAC Workshop on Robust Control Design*, (Haifa, Isreael), pp. 337–342, 2009.

[2] C. Roos, "Generation of flexible aircraft LFT models for robustness analysis," in *6th IFAC Workshop on Robust Control Design*, (Haifa, Isreael), pp. 349–354, 2009.